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## **Didactic proposal for calculation of areas between two curves with the support of GeoGebra**

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#### **Abstract**

The research problem is related to learning the topic of calculating the area between two curves with the support of GeoGebra. This research was quasi-experimental, since it worked with two previously established groups, called control and experimental, respectively. The objective was to determine the effect produced by the application of a didactic proposal, based on the theoretical framework of Duval's semiotic representations that includes the use of GeoGebra, on the results of student learning. It was a quantitative research by comparing the learning results by means of a post-test and an opinion survey applied to the students of the experimental group. According to the results of the statistical analysis, it is concluded that with the application of the didactic proposal the students of the experimental group obtained better learning results than those of the control group.

**Keywords:** Learning, calculation of areas, GeoGebra, semiotic representations

#### **Resumen**

El problema de investigación está relacionado con el aprendizaje del tema del cálculo de área entre dos curvas con apoyo de GeoGebra. Esta investigación fue de tipo cuasi-experimental, ya que se trabajó con dos grupos previamente establecidos, denominados de control y experimental, respectivamente. El objetivo fue el de determinar el efecto que produce la aplicación de una propuesta didáctica, basada en el marco teórico de las representaciones semióticas de Duval que incluye el uso GeoGebra, sobre los resultados de aprendizaje de los estudiantes. Fue una investigación de tipo cuantitativo al comparar los resultados de aprendizaje por medio de un postest y de una encuesta de opinión aplicada a los alumnos del grupo experimental. De acuerdo con los resultados del análisis estadístico, se concluye que con la aplicación de la propuesta didáctica los alumnos del grupo experimental obtuvieron estadísticamente mejores resultados de aprendizaje que los del grupo control.

**Palabras clave:** Aprendizaje, Cálculo de áreas, GeoGebra, representaciones semióticas.

## **Introduction**

### **Problematic situation**

In recent times, technological development has shown considerable progress, some of its advances have had an impact on education and have become an instrument that supports both students and teachers. With such advances, it is possible to access specialized digital tools, many of them low-priced, such as free software, which makes it possible to experiment, in a fast way, those topics that involve problems either graphically or analytically, also to making calculations that in a traditional way would be very heavy.

The use of Information and Communication Technologies (ICT) provide an important role in educational development because it makes possible to use different strategies with which they can establish new ways both teach and learn.

For this research, Duval's semiotic representation registers were used as a reference (Duval, 1998) and its relevance in learning, applying the transformations of the representation within the same register (treatment) and the conversion which is the change between semiotic representations (Penalva & Torregrosa, 2017). One of the main results obtained was that the students were able to identify different forms of representation and move through them, which favored the conceptualization of related concepts.

As for the part of the topic of teaching the definite integral of a variable, which is where the calculation of the area between two curves is shown, Contreras & Ordóñez (2006) affirm that the research related to this topic has been associated with two fundamental factors: the analysis of the difficulties of understanding the students of the concepts, where has a great implication the mathematical object of limit, and on the other hand, the bias regarding the use of the definite integral towards a certain exclusivity of the algorithmization in the calculation, this has led to an algebraic sense of the very distant concept of its interpretation as a result of the change process.

About it Artigue (2002) mentions the limitations on the conduct research considering statistical approaches and the importance of quantitative studies in didactics of mathematics. For this reason, the methodology of the present investigation was quantitative as a result of the same.

## **Antecedents**

### ***Duval's semiotic representations***

The treatment and conversion of representation registers are fundamental activities for the understanding of mathematical objects, so that, for learning calculation area between two curves it was used as theoretical reference the registers of Duval's semiotic representations.

The main thesis that characterizes Duval's model of semiotic systems of representation (Duval, 2003), establishes that there can be no understanding in mathematics if a mathematical object is not distinguished from its representation.

The representation of a concept is the most elementary of the cognitive functions originated in mathematical activity, so that, in the construction of a mathematical concept, in addition to the representation, it is necessary to carry out a treatment of the object within the same register (for example, between different algebraic forms), and additionally reach the conversion between the different registers, considered as a fundamental activity in the acquisition of a concept. The mediation of different semiotic registers is not only necessary to express a mathematical idea, but they are a constitutive part of it (Duval, 2006).

In Duval's theoretical framework, semiotic representations are defined as productions made by the use of signs that belong to a representation system, which has its own limitations of meaning and operation (Duval, 2003). It is affirmed that semiotic representations are absolutely necessary for the mathematical activity because its objects can not be seen directly and therefore should be represented.

Semiotic representations have a fundamental role in the study of mathematics, since, unlike other areas of study, mathematical objects are not tangible and are difficult to visualize, this makes these representations through the concept becomes accessible. According to Duval (2003) it is important for the student to differentiate the mathematical object from its representations, since otherwise it is not possible that conceptualizes.

Duval (2003, p. 3) argues that “in mathematics, semiotic representations are not only indispensable for communication purposes, but they are also essential for the development of the

mathematical activity itself”; and makes a difference between semiosis what is the mental production of a representation and the noesis which is the conceptual understanding of an object, in addition to maintaining the assumption that they are inseparable since for him there is no noesis without semiosis.

Duval (2006) distinguishes three types of cognitive activities linked to semiosis:

- 1) The formation of a representation that can be identified as belonging to a specific register. The representations of a particular semiotic register, must comply with rules of conformity, for reasons of communication of representations called concept formation.
- 2) The processing or treatment that is the transformation of a representation in the same register where it was created. It is called treatment of a representation, to the transformations of the representation within the same register. It is formed according to the rules that are specific to the system; so that from these other representations are obtained that can be constituted as a gain of knowledge in comparison with the initial representations. Because of this, each treatment requires the recognition and application of own rules of each register. In the particular case of the proposal, the mental treatment operation is carried out, for example, in exercise 6 of the workbook, when the area is calculated using an algebraic procedure given a particular integrated iterated.
- 3) The conversion of a semiotic representation from one register to another. It is called conversion to the ability to change the representations produced from one representation system to another, so that this other system allows explicit other meanings relating to what is represented (Ospina, 2017). For example, move from an algebraic geometric representation, ie, graph an equation.

Also, Duval (2006) notes that the importance of conversion describing it as a necessary passage for coordination between registers of the same concept. The possibility of conversion between registers of semiotic representation is an essential condition for the conceptualization and, therefore, of the conceptual understanding of an object (noesis).

While mental representations they are formed of the set of conceptions that an individual has about a mathematical object, semiotic representations are productions constituted by the use of signs, are the means to externalize mental representations.

For Duval (2003), the importance of using different representational registers in understanding a concept is:

- The economy of treatment, since it can happen that the relationships between mathematical objects can be represented in a faster and simpler way in some of the registers.
- The complementarity of registers, then the aspects of the content of a situation, are not the same from one register to another in which they are represented.
- And the conceptualization, involved in the coordination of representation registers.

Ultimately, from the perspective of Duval's theoretical model, the acquisition of mathematical concepts in an individual will occur at the moment in which there is a coordination free of contradictions in, at least, two different registers of semiotic representation of the object. In analyzing the use of semiotic representation registers, the recognition of the register should be taken into account, the transformations inside it (treatment) and conversion between them (Duval, 1998).

In this research, the register of graphic and algebraic representation were essentially considered. In the graphical representation register, the student constructs geometric figures or draws regions in a Cartesian coordinate system in understanding the calculus of areas with or without the help of GeoGebra. On the other hand, in the algebraic representation register, the student operates and solves definite integrals and raises inequalities to refer and calculate the area between two curves.

## **Objective**

The research was conducted with students from the University Center of Exact Sciences and Engineering (CUCEI) of the third semester of the Electrical Mechanics career, whose objective was to determine the effect produced by the application of a didactic proposal, based on Duval's theoretical framework of semiotic representations, which includes the use of

GeoGebra (Hohenwarter, 2015), on the results of student learning in the topic area between two curves.

For the aforementioned, the research problem is related to the learning of the topic of calculating the area between two curves with the support of the free software GeoGebra.

Also, the research focused on answering the following research questions:

How the didactic proposal for calculating areas between two curves with the support of GeoGebra software influences the learning results of students?

How the activities in the workbook, designed for learning the topic of calculation of areas between two curves, influence the learning outcomes of the topic?

How are the learning results on the topic of calculation of areas between two curves of the experimental group, compared to the control group?

What do the students think about the use of the didactic proposal in the area of calculation of areas between two curves and the teacher's support?

The topics covered in the didactic proposal are part of the official thematic content of the program of the Advanced Calculus course. These topics are seen in unit 4 Multiple Integrals, in particular in topics 4.1 Iterated integrals and 4.2 Integrals over flat regions.

## **Materials and method**

### **Participants**

The experimentation was carried out with two groups of third semester students of the Electrical Mechanical Engineering career of the MT113 Advanced Calculus course, whose program includes the topic of the definite integral and in particular that of the area calculation between two curves. One of the groups was called a control group and the other experimental group, the control group, was conformed of 39 students, while the experimental group was conformed of 32 students.

Each group was attended by the professor assigned to each of them. The control group traditionally worked, ie, used only conventional resources like white board and marker; while the experimental one was applied the didactic proposal of the topic of calculation of area between



two curves, based on the theoretical framework of Duval's semiotic representations that includes the use of GeoGebra.

### **Technique and instruments**

To conduct the research, they reviewed the antecedents of the topic and different materials were developed.

A workbook was developed with learning activities based on Duval's theoretical model of semiotic representations, which also included some exercises in which ICT were used, particularly with support from the GeoGebra program so that students could define the integration limits of regions limited by some graphs in  $R^2$ . The workbook was applied exclusively to the experimental group.

Subsequently, a post-test was made, in which it is described in the corresponding section, according to the contents and objectives of the course Advanced Calculus, respect the topic of calculating the area between two curves and this instrument was applied to the two groups to measure the effect of the treatment in the experimental group in comparison with the results of the control group.

Another instrument that was applied to the experimental group was an opinion survey, which contains questions about the students' considerations respect the development of the activities in the workbook, the use of the GeoGebra program, the teacher's performance and also a section for comments and suggestions.

### **Workbook**

It was the instrument that contained the learning activities of the didactic proposal, which was answered individually by the students of the experimental group and the elaboration of the sequence of activities was based on Duval's theoretical model of semiotic representations, with GeoGebra support. It consists of nine activities which are described below:

1. Given the functions plot area regions.
2. Using GeoGebra.
3. Dada las gráficas determinar los límites de integración verticalmente simple.
4. Given the graphs, determine the limits of horizontally simple integration.



5. Given the limits of integration of a region  $R$ , propose the iterated integral that calculates the area.
6. Determination of the area given an iterated integral.
7. Given the graphs of the area of the region and its functions define limits of integration and pose the iterated integral.
8. Given the regions between two curves, in their graphical and algebraic representations, calculate the area in either of the two integration orders.
9. Change the order of integration of an iterated integral.

To guide the teaching of the topic of calculation of areas between two curves, each of the students was given a copy of the workbook, containing definitions and exercises in the registers of graphic and algebraic representation, some examples solved with procedures, as well as unresolved practice exercises to be developed inside and outside the classroom.

Also, the workbook was used as instructional material to guide the students in order to carry out a sequence that allowed them:

1. represent an area or region in one of the register of semiotic representation (formation) and
2. relate different representations in the same register (treatment) or
3. from one register to another (conversion).

For this, in different activities the use of the GeoGebra program was incorporated as support to represent the area of an integration region. It is worth mentioning, that the program has different versions and the students could choose to use it either on their computer or on a cell phone.

### ***Post-test***

The purpose of the post-test was to evaluate the learning result of the students on the topic of calculating the area between two curves. This instrument consisted of seven exercises where the following were requested:

1. Pose the iterated integral that calculates the area of the region between the graphs.
2. Draw the area of the equations and pose the iterated integral.

3. Given the equations and graph pose and solve the iterated integral.
4. Given the equations calculate the area of the region.
5. Given the equations, graph, pose and solve the iterated integral.
6. Change the order of integration of the iterated integral.
7. Change the order of integration of the iterated integral and plot its area region.

In Figure 1 shows the first three exercises, while in Figure 2 the remaining are presented.

Figure 1.

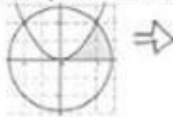
First part of the post-test.

**Postest**  $\frac{11}{11}$   
**Cálculo de áreas**

Nombre: \_\_\_\_\_

1. Plantear una integral iterada que calcule el área de la región comprendida entre las gráficas de las ecuaciones  $x^2 + y^2 = 2$ ,  $y = x^2$  y  $y = 0$ , en el primer cuadrante.

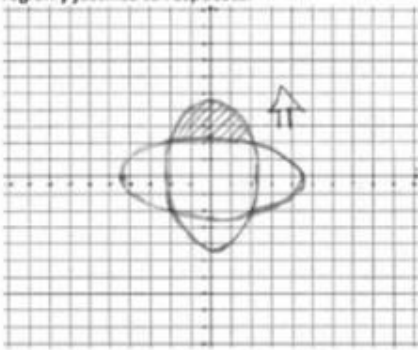
$x = \sqrt{2-y^2}$     $x = \sqrt{y}$



a)  $\int_0^{\sqrt{2}} \int_{\sqrt{2-y^2}}^{\sqrt{y}} dx dy$    b)  $\int_0^1 \int_{x^2}^{\sqrt{2-x^2}} dy dx$    c)  $\int_0^1 \int_{\sqrt{y}}^{\sqrt{2-y^2}} dx dy$    d)  $\int_0^{\sqrt{2}} \int_{\sqrt{2-y^2}}^{\sqrt{y}} dy dx$

2. Plantear una integral iterada que calcule el área interior a  $\frac{x^2}{5} + \frac{y^2}{20} = 1$  y superior a  $\frac{x^2}{20} + \frac{y^2}{5} = 1$ . Traza la región y justifica tu respuesta.

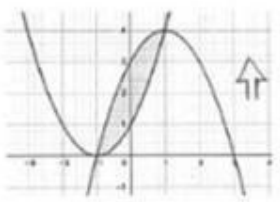
$y = \sqrt{20-x^2}$     $y = \sqrt{5-\frac{x^2}{4}}$



$\int_{-2}^2 \int_{\sqrt{5-\frac{x^2}{4}}}^{\sqrt{20-x^2}} dy dx$

Integral: \_\_\_\_\_

3. Utiliza una integral doble para calcular el área de la región comprendida entre las ecuaciones  $y = -(x-1)^2 + 4$  y  $y = (x+1)^2$ . Justifica tu respuesta.



$\int_{-1}^1 \int_{(x+1)^2}^{-(x-1)^2+4} dy dx$

$\int_{(x+1)^2}^{-(x-1)^2+4} dy = y \Big|_{(x+1)^2}^{-(x-1)^2+4} = [-x^2+2x+5] - [x^2+2x+1]$

$= -x^2+2x+5-x^2-2x-1 = -2x^2+4$

$\int_{-1}^1 -2x^2+4 dx = -\frac{2}{3}x^3+4x \Big|_{-1}^1$

Área:  $\frac{8}{3} \approx 2.6667$

$[(\frac{2}{3}+4)] - [(-\frac{2}{3}-4)] = [\frac{14}{3}] - [-\frac{14}{3}] = \frac{28}{3} \approx 9.3333$

$(x+1)(x+1) = x^2+2x+1$

$-(x-1)(x-1)+4 = -(x^2-2x+1)+4$

$= -x^2+2x-1+4$

$= -x^2+2x+3$

Source: Scanned image of an exam.

Figure 2.

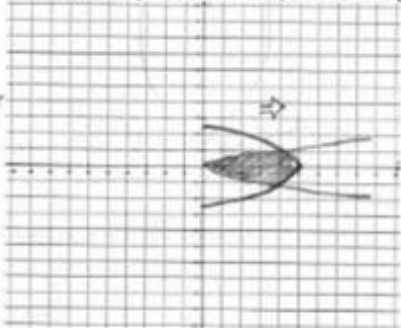
Remaining exercises of the post-test.

$\int_0^1 \int_{\sqrt{x}}^{x^2+1} dy dx = \frac{2}{3}$

4. Calcular el área de la región comprendida entre las ecuaciones  $y = x^2 + 1$ ,  $y = \sqrt{x}$ ,  $x = 0$  y  $x = 1$ .

a) 2      b)  $\frac{5}{6}$       c)  $\frac{4}{3}$       d)  $\frac{2}{3}$       e) No tiene solución

5. Traza la región acotada por las ecuaciones  $x = -y^2 + 5$  y  $x = 4y^2$ . Expresa el área de la región como una integral doble iterada y evalúa la integral para definir el área. Justifica tu respuesta.



Integral:  $\int_{-1}^1 \int_{4y^2}^{-y^2+5} dx dy$

$\int_{4y^2}^{-y^2+5} dx = x \Big|_{4y^2}^{-y^2+5} = -y^2 + 5 - 4y^2$

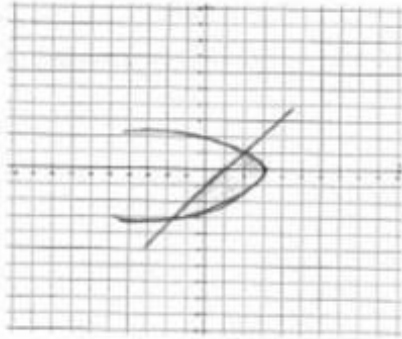
$\int_{-1}^1 (-y^2 + 5 - 4y^2) dy = \int_{-1}^1 (-5y^2 + 5) dy = \left[ -\frac{5}{3}y^3 + 5y \right]_{-1}^1 = \left[ -\frac{5}{3} + 5 \right] - \left[ \frac{5}{3} - 5 \right] = \left[ \frac{10}{3} \right] - \left[ -\frac{10}{3} \right] = \frac{20}{3}$

Integral:  $\int_{-1}^1 \int_{4y^2}^{-y^2+5} dx dy$  Área:  $\frac{20}{3} \approx 6.666$

6. Cambia el orden de integración de la integral  $\int_0^4 \int_0^{\sqrt{x}} dy dx$

a)  $\int_0^4 \int_{y^2}^2 dx dy$       b)  $\int_0^2 \int_0^{\sqrt{y}} dx dy$       c)  $\int_0^2 \int_{y^2}^4 dx dy$       d)  $\int_0^4 \int_0^{\sqrt{y}} dx dy$       e)  $\int_0^2 \int_0^{y^2} dx dy$

7. Cambia el orden de integración de la integral  $\int_{-2}^1 \int_{y+1}^{3-y^2} dx dy$ . Traza la región de integración y justifica tu respuesta.



Integral:  $\int_{-1}^2 \int_{y+1}^{3-y^2} dx dy + \int_2^3 \int_{x-1}^{\sqrt{3-x}} dy dx$

Integral: \_\_\_\_\_

$x = 3 - y^2$        $y = \pm\sqrt{3-x}$   
 $x = y + 1$        $y = x - 1$

Source: Scanned image of an exam.

**Opinion survey**

At the end of the post-test, an opinion survey was applied to the experimental group (see appendix). The measurement of it was carried out using a Likert scale and was designed in order

to know the opinion of the students respect the development of the activities in the workbook, the use of the GeoGebra program, the teacher's performance and also a section for comments and suggestions. The values and criteria of the Likert scale for the opinion survey are shown in Table 1, while the questions are presented in Table 2.

Table 1.

*Values and criteria for the answers to the questions.*

Value	1	2	3	4	5
Criteria	Completely disagree	Disagree	Is indifferent to me	Agree	Completely agree

*Source:* own elaboration.

Table 2.

*Opinion survey questions.*

Num.	Question
1	The vision of the topic was improved with the development of the activities.
2	The activities carried out were interesting.
3	The use of technology motivates interest in learning the topic.
4	The use of the programs facilitates the interpretation of graphs.
5	The time dedicated to the activities you consider was sufficient.
6	You consider that the methodology used in each of the sessions was adequate.
7	I like doing the activities in the workbook.
8	The instructions in the workbook were clear and understandable.
9	The use of the workbook was clear and structured.
10	The activities carried out encouraged the learning of the topic.
11	With the activities the topic was covered according to the thematic content.
12	The support provided by the teacher was adequate.
13	The support provided by the teacher was adequate.
14	The teacher's performance for the development of the topic was adequate.
15	The teacher managed to develop a positive work environment (respect, openness to dialogue, feedback).
16	You consider that, in general, the material used for the topic was of quality.

*Source:* own elaboration.

Finally, the comments and suggestions section consisted of three open questions:

- From what you learned in class on the topic of calculating areas between two curves, what aspects they called your attention?
- In general, give your opinion on the activities carried out, the use of the software and the performance of the teachers.
- Write a comment that is not covered in the previous questions and that is related to the quality of the material used or the way it was presented or used.

## Process

The experimentation lasted 8 hours, divided into four sessions of two hours each. In Table 3 briefly describes the activities carried out in each session.

Table 3.

*Sessions and activities of the experimental phase.*

Session	hours	Description
1	2	In this session, the workbook was given in printed form to the students who were going to participate in the experiment. Immediately, its content was analyzed and the corresponding instructions were given to answer it appropriately. Finally, the students were asked to bring a computer or cell phone in the next session to provide them with the files necessary for experimentation.
2	2	In the first half hour, the GeoGebra files that would be used in the experimentation were delivered. In the remaining time, the resolution of the activities proposed in the workbook began verbally, visually supporting the files created in GeoGebra. In this session the first four activities were covered, of a total of nine, of the workbook.
3	2	In this session the five remaining activities of the workbook were completed.
4	2	In the first hour and a half the post-test was applied and in the remaining half hour the opinion survey was answered by the students.

*Source:* own elaboration.

## Results and discussions

### Analysis of the learning results achieved by the students

The post-test questions were presented in a format designed for the student to complete or answer the questions requested. Each question was evaluated with the criteria as shown in Table 4.

Table 4.

*Post-test evaluation criteria.*

Question	Criteria	Evaluation
1	Pose the iterated integral	1 point if it is correct 0 point incorrect
2	Draw the area of the equations and pose an iterated integral	2 points correct solution 1 point incomplete solution 0 point incorrect solution
3	Graph, pose, and solve an iterated integral	2 points correct solution 1 point incomplete solution 0 point incorrect solution
4	Calculate the area of a region	1 point if it is correct 0 point incorrecta
5	Graph, pose and solve an iterated integral	2 points correct solution 1 point incomplete solution 0 point incorrect solution
6	Change the order of integration of an iterated integral	1 point if it is correct 0 point incorrecta
7	Change the integration order of an integral and plot its area region	2 points correct solution 1 point incomplete solution 0 point incorrect solution

*Source:* own elaboration.

In Table 5 shows the data of the results of the responses of the students in the post-test to the control group and in Table 6 those corresponding to the experimental group.



Table 5.

*Post-test results corresponding to the control group.*

Student	Question							Average
	1	2	3	4	5	6	7	
1	0	0	0	0	0	0	0	0.00
2	1	2	1	1	2	1	1	81.82
3	1	2	1	1	2	0	0	63.64
4	1	2	2	1	2	0	2	90.91
5	0	0	0	0	0	0	0	0.00
6	1	1	2	0	2	0	2	72.73
7	1	0	0	1	1	0	1	36.36
8	1	0	0	0	2	1	0	36.36
9	1	2	2	1	2	1	1	90.91
10	1	1	2	1	2	1	2	90.91
11	0	0	2	1	0	1	0	36.36
12	1	2	1	0	2	1	2	81.82
13	1	2	2	1	1	1	2	90.91
14	1	1	0	1	2	1	1	63.64
15	0	1	2	1	1	1	1	63.64
16	1	0	1	0	1	1	2	54.55
17	1	2	1	1	2	0	1	72.73
18	1	2	2	1	2	1	2	100.00
19	1	0	1	1	1	1	2	63.64
20	1	0	1	1	1	0	0	36.36
21	1	0	1	1	1	1	2	63.64
22	1	0	0	1	1	0	1	36.36
23	1	0	2	0	2	0	0	45.45
24	1	1	0	1	1	1	2	63.64
25	1	0	0	1	0	0	0	18.18
26	0	1	1	1	2	1	1	63.64
27	1	0	1	1	2	1	1	63.64
28	1	2	2	1	1	1	2	90.91
29	1	1	2	0	2	1	2	81.82
30	1	0	2	1	2	0	0	54.55
31	1	1	0	0	0	1	0	27.27
32	1	0	2	1	1	1	2	72.73
	84.38	40.63	56.25	71.88	69.35	62.50	54.69	63.64

Source: own elaboration.

Table 6.

Post-test results corresponding to the experimental group.

Student	Question							Average
	1	2	3	4	5	6	7	
1	1	2	2	1	2	1	0	81.82
2	1	2	2	1	2	1	2	100.00
3	0	2	2	1	2	1	2	90.91
4	1	2	2	1	2	1	1	90.91
5	1	0	2	1	2	1	1	72.73
6	1	2	2	1	2	1	2	100.00
7	1	2	2	1	2	1	2	100.00
8	1	2	2	1	2	0	2	90.91
9	1	2	2	1	2	1	1	90.91
10	1	1	2	1	2	1	2	90.91
11	1	2	2	1	2	1	1	90.91
12	1	2	1	1	2	1	1	81.82
13	1	2	2	1	2	1	1	90.91
14	1	0	2	1	2	0	1	63.64
15	1	2	2	1	2	1	2	100.00
16	1	2	1	1	2	1	1	81.82
17	1	2	2	1	2	0	2	90.91
18	1	2	2	1	2	1	1	90.91
19	1	0	2	1	2	1	1	72.73
20	1	2	2	1	2	0	2	90.91
21	1	2	2	1	2	1	2	100.00
22	1	2	1	1	2	1	2	90.91
23	1	1	2	1	1	1	1	72.73
24	1	2	2	1	2	1	2	100.00
25	1	2	2	1	2	1	2	100.00
26	1	2	2	1	2	1	2	100.00
27	1	2	2	1	2	1	2	100.00
28	0	0	1	1	1	1	0	36.36
29	1	2	2	1	2	1	2	100.00
30	1	2	2	1	2	1	2	100.00
31	0	2	2	1	2	1	2	90.91
32	0	2	2	1	2	1	2	90.91
33	1	2	2	1	2	1	2	100.00
34	1	1	2	1	2	1	1	81.82
35	0	2	2	0	1	0	1	54.55
36	1	2	2	1	2	1	2	100.00
37	0	2	2	1	1	1	1	72.73
38	1	2	2	1	2	1	2	100.00
39	1	2	2	1	2	1	2	100.00
	84.62	85.90	94.87	97.44	94.87	87.18	76.92	88.35

Source: own elaboration.

### ***Statistical results of the post-test***

In Table 7 shows a statistical summary of the results of the application of the post-test of the control and experimental groups carried out with the statistical software Statgraphics® (Polhemus, 2014), since it presents the results in an understandable way and does not use programming, where it is shown that the average of the experimental group was higher than that of the control group; also, given that the results show that the standard deviation, which is a measure of group variability, is lower in the group where the didactic proposal was applied; this group is considered to be more homogeneous. It should be mentioned that in the control group two students did not present the post-test, so they were not considered in the statistical analysis.

Table 7.

*Statistical summary of the post-test.*

	Control group	Experimental Group
Count	30	39
Average	63.6364	88.345
Standard deviation	21.8808	14.1373
Minimum	18.1818	36.3636
Maximum	100.0	100.0
Rank	81.8182	63.6364

*Source:* own elaboration.

In Table 8 shows that the confidence interval for the difference between the means, which ranges from -33.3875 to -16.0297, and since the interval does not contain the value 0, the existence of a statistically significant difference is confirmed between the means of the two samples, with a confidence level of 95.0%.

Table 8.

*Averages and confidence intervals.*

Group	Mean	Confidence intervals 95%
Control	63.63	55.46 – 71.80
Experimental	88.34	83.76 – 92.92
Confidence interval 95.0% for the mean difference:		-33.3875 – -16.0297

Source: own elaboration.

It was carried out a comparison of the means of control and experimental group group, with the test statistic *t-student*, with a significance level  $\alpha = 0.05$ . This test was carried out with the statistical software Statgraphics® (Polhemus, 2014), for the comparison of the means, a

$$t = -5.68259$$

to which corresponds a probability value

$$P = 0.000000528634$$

than compared to the value of  $\alpha = 0.05$ , you have the value of  $P < \alpha$ , and therefore the null hypothesis regarding the equivalence of the group means is rejected.

Specifically, the *t-student* test was used to evaluate hypotheses about the difference between the means:

Null hypothesis: With the use of the didactic proposal, the learning results of the experimental group are not statistically better than the learning results of the control group.

$$H_0: \bar{X}_E \leq \bar{X}_C$$

Alternative hypothesis: With the use of the didactic proposal, the learning results of the experimental group are statistically better than the learning results of the control group.

$$H_A: \bar{X}_E > \bar{X}_C$$

where:  $\bar{X}_E, \bar{X}_C$ , represent the sample mean of the experimental and control groups respectively.

The test was constructed to determine if the difference between the two means was equal to 0 against the alternate hypothesis that the difference was not equal to 0. Since the calculated P-value was less than 0.05, the null hypothesis could be rejected in favor of the alternative hypothesis, ie, that with the application of the didactic proposal, better learning results of the area calculation topic between two curves were obtained by the students of the experimental group, compared to the control group that received a traditional treatment.

### Analysis of the opinion survey

In Table 9 shows the percentages of occurrence in each of the questions of the opinion survey applied to the experimental group according to the established criteria.

Table 9.

*Percentages of each question according to the established criteria.*

Question	Criteria				
	Completely disagree	Disagree	Is indifferent to me	Agree	Completely agree
1	76.92	15.38	7.69	0.00	0.00
2	61.54	33.33	5.13	0.00	0.00
3	76.92	17.95	2.56	2.56	0.00
4	87.18	12.82	0.00	0.00	0.00
5	56.41	23.08	5.13	12.82	2.56
6	48.72	38.46	12.82	0.00	0.00
7	56.41	41.03	2.56	0.00	0.00
8	53.85	38.46	5.13	2.56	0.00
9	61.54	38.46	0.00	0.00	0.00
10	66.67	28.21	5.13	0.00	0.00
11	69.23	23.08	7.69	0.00	0.00
12	56.41	28.21	12.82	2.56	0.00
13	48.72	41.03	7.69	0.00	2.56
14	61.54	33.33	5.13	0.00	0.00
15	74.36	17.95	5.13	2.56	0.00
16	79.49	20.51	0.00	0.00	0.00

*Source:* own elaboration.

Considering the values for the established criteria, in Table 10 shows these values for the responses to each of the opinion survey questions by the students.

Table 10.

*Answers to opinion questions on a Likert scale.*

Student	Question															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	5	4	4	5	5	4	4	4	4	4	5	4	4	4	4	4
2	5	4	4	5	5	4	4	4	5	5	4	5	5	5	5	5
3	5	4	5	5	4	4	4	4	5	5	4	5	4	4	5	5
4	4	4	5	5	4	4	4	4	4	4	5	4	4	4	4	5
5	5	4	4	5	2	4	5	5	5	5	5	3	4	4	3	4
6	4	5	5	5	2	4	4	4	4	4	4	3	4	4	4	4
7	5	4	4	4	4	4	4	4	4	5	4	4	4	4	4	4
8	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
9	5	5	4	5	5	5	5	5	5	5	5	5	5	5	5	5
10	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
11	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
12	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
13	4	4	5	4	3	3	4	4	4	4	3	4	3	3	4	4
14	5	5	5	5	5	4	5	5	5	5	5	5	5	5	5	5
15	3	4	2	5	5	3	4	4	4	3	5	5	5	4	4	5
16	4	5	5	4	4	4	4	4	4	4	4	4	4	5	5	5
17	5	4	4	5	5	4	4	5	4	5	5	5	4	4	3	4
18	5	5	5	5	5	5	5	5	4	4	5	4	4	4	5	5
19	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
20	5	4	5	5	5	5	5	5	5	5	5	4	5	5	5	5
21	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
22	5	5	5	5	5	5	5	5	5	4	4	5	4	4	5	5
23	5	4	3	5	4	4	3	4	4	5	4	4	4	4	5	5
24	3	3	5	5	1	3	4	3	5	3	3	3	3	4	5	5
25	5	5	5	5	4	5	5	4	5	5	5	5	5	5	5	5
26	5	4	5	5	2	4	4	2	4	5	5	5	5	5	5	5
27	3	5	5	5	3	3	4	3	4	4	3	2	1	3	2	4
28	5	5	5	4	4	5	4	4	4	4	5	5	5	5	5	5
29	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
30	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
31	4	3	5	5	2	3	5	5	5	5	5	5	4	5	5	5
32	5	4	5	5	5	5	5	4	5	5	5	4	4	5	5	5
33	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5	5
34	5	5	5	5	2	4	5	5	5	5	5	5	5	5	5	5
35	5	5	5	5	5	5	5	5	5	5	5	4	3	4	5	5
36	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
37	5	5	5	5	5	5	5	5	5	5	4	3	4	5	5	5
38	4	5	5	4	4	4	4	4	4	4	5	4	4	5	5	5
39	5	5	4	5	4	4	4	4	5	4	4	3	4	5	4	4

Source: own elaboration.

From the students' responses, it is observed that more than 90% consider that with the activities designed in the didactic proposal the vision of the topic was improved, that they were

interesting; The interpretation of the graphs was facilitated, their interest in learning the subject was motivated and fostered, the thematic content was covered, which they liked to carry out in a workbook in addition to the fact that their instructions and methodology were clear and precise, in addition to the fact that the material used was of quality.

It should be noted that with the use of the computer and the cell phone, the students showed interest in the subject, because by entering the equations, they were able to visualize and manipulate the area of the region to be calculated, and also define the integration limits. The students pointed out that the use of GeoGebra, also to the speed to graph, facilitated the exploration to interpret the region of area bounded by the functions. Also, the structure of the workbook activities was designed according to the advanced calculation program. The difficulty level was the simple to the complex considering the different registers of semiotic representation.

With regard to the time spent on activities, approximately 20% of students felt that they did not seem enough. In this case, during the elaboration of the exercises in the workbook, if the time to complete the programmed activity was not enough, they were asked to deliver it as homework in the next class.

Also, More than 90% of the students considered that the teacher's support and performance was adequate, that his instructions were clear, and that a positive work environment was achieved, although it seemed to one student that they were not entirely so.

During the application of the experiment, in each of the sections of the workbook, it was intended to indicate the instructions as clearly as possible, however, there were some students who did not understand them and it was explained to them what was required to complete or solve the exercise and with that support them so that they understood what was requested, in addition to that at all times they were attending to the doubts that were presented.

With regard to the comments section and suggestions, student responses are summarized below:

- From what you learned in class on the topic of calculating areas between two curves, what aspects caught your attention the most?



Most of the students found it interesting to be able to verify, by means of graphical and algebraic representations, that the calculation of a flat region can be defined by the limits of integration in the vertical or horizontally simple order.

- In general, give your opinion on the activities carried out, the use of the software and the performance of the teachers.

As for the activities commented that they were well raised, besides being interesting area calculation by iterated integrals. Regarding the use of GeoGebra, the students considered that it made it easier for them to learn the topic, understand and visualize the area to be calculated. The instructor's explanation supported the efficient use of the workbook, the interaction with the GeoGebra software, the interpretation of the graphs, and the area calculations that the students performed.

- Write an extra comment about the printed material (workbook) used.

Most of the students commented that they liked the work methodology proposed in the workbook, especially regarding the use of the software, and they also suggested including the solutions to the proposed exercises.

It was considered that the use of the software turned out to be a quick and effective tool for the students to become familiar with the subject by relating the algebraic with the geometric representations, and that it helped them to explore and interpret the calculation of the area of the regions with which they they were asked to work.

### **Answer to research questions**

Coming up next, the research questions are answered from the qualitative and quantitative analysis that was carried out as a result of the application of the didactic proposal for learning the area between two curves.

### ***How the didactic proposal for calculating areas between two curves with the support of GeoGebra software influences the learning results of students?***

Based on the interpretation of the data analysis, where a quantitative evaluation of the results of the post-test exam was applied, a positive effect was found on the learning of the experimental group, from the application of the didactic proposal of the subject of calculation of

areas between two curves, which is supported by the results obtained through a hypothesis test in which the means of the control group and the experimental group were contrasted.

It was verified that there was a statistically significant difference between the means of both groups, when performing an analysis with the statistical software Statgraphics® (Polhemus, 2014), in which the *t-student* test statistic was used, a probabilistic value was obtained  $P = 0.000000528634$ , less than the significance level of  $\alpha = 0.05$ , and therefore the learning of the experimental group of the subject of calculation of areas between two curves was better compared to the control group.

***How the activities in the workbook, designed for learning the topic of calculation of areas between two curves, influence the learning outcomes of the topic?***

In the opinion survey carried out, it was found that the majority of the students (76.92%), affirmed that the activities carried out improved the perspective on the subject calculation of areas between two functions, 15.38%, six students agreed and only three students 7.69% were indifferent. Also, they noted that the use of the GeoGebra program, in addition to achieving greater efficiency in graphing the regions, facilitated the exploration of the most convenient integration order for the calculation of the area bounded by the functions. 66.67% of the students, that is, 26 students, fully agreed that the activities carried out promoted the learning of the exposed topics, 11 of them (28.21%) agreed and only 2 (5.13%) were indifferent.

***How are the learning results on the topic of calculation of areas between two curves of the experimental group, compared to the control group?***

The learning results of the area calculation topic between two curves by the experimental group were statistically better compared to those of the control group. From the averages obtained in the post-test, it was possible to verify with 95% confidence, by means of a t-student test for the comparison of the group means, that the calculated value of  $P = 0.000000528634$  compared to the significance value of  $\alpha = 0.05$ , that  $P < \alpha$ , and therefore the null hypothesis was rejected regarding that with the use of the didactic proposal the learning results of the experimental group would not be better than the learning results of the control group, ie, in the experimental group better learning results were obtained.

Also, it was shown that the experimental group was more homogeneous, given that its standard deviation (14.1373) was lower than the control group (21.8808), in fact using an  $F$ -test to evaluate the hypothesis about the quotient of the standard deviations was equal to 1.0, compared to the alternative hypothesis that the quotient was not equal to 1.0, a calculated  $P$  value less than 0.05, then, the null hypothesis could be rejected, verifying that there was a statistically significant difference between the standard deviations of the control and experimental groups.

***What do the students think about the use of the didactic proposal in the area of calculation of areas between two curves and the teacher's support?***

As expressed by the students in the survey, the majority (76.92%) were motivated due to the use of technology for the determination and calculation of areas between curves, 7 students (17.95%) agreed, to one student (2.56%) was indifferent and another 2.56% disagreed.

56.41% of the students (22) were enthusiastic about the activities corresponding to the workbook, 16 of the students (41.03%) agreed and only 2.56% (1) were indifferent.

The support offered by the teacher was adequate for 56.41% (22) of students who fully agreed, 28.21% (11) of students agreed, 12.82% (5) were indifferent and only 2.56% (1) in disagreement.

The opinion of the students about the proposal in general is that the activities were clearly raised, also to considering that they were interesting and that the use of ICT facilitated the learning of the area of calculation between two curves through iterated integrals.

The effect produced by the didactic proposal based on Duval's semiotic representations was positive in the experimental group, because when carrying out the activities with the guide of the workbook, with their planned and structured tasks, have software support and perform graphical explorations of the functions that were analyzed, led them to make the conversions that helped them to visualize and carry out the corresponding calculations; also, was positive the support offered to the teacher clarify their doubts at the time. The results of the statistical analysis confirm the above.

## **Conclusions**

Based on the results of the statistical analysis, it is concluded that there is a significant difference in the learning of the experimental group compared to that of the students of the control group, who received traditional instruction, ie, that with the application of the didactic proposal, better learning results were obtained in the students of the experimental group of the third semester of Electrical Mechanical Engineering in the topic MT113 Advanced Calculus from CUCEI, in the area calculation of areas between two curves.

For the students, as expressed in the opinion survey that was applied to them at the end of the treatment, The activities related to the topic of calculating the area between two curves with the use of the GeoGebra program were interesting, which it caused to a good participation of the students. It should be noted that the proposal was applied when the course had already started for nine weeks, therefore, the students already had some background on the topic, and by proposing to use ICT they were motivated and agreed to participate.

The material that was used in the didactic proposal helped to identify the different semiotic representations of a particular record and the conversion of the produced representations from one representation system to another, as mentioned in Ospina (2017) and Penalva & Torregrosa (2017). In other words, the students were able to interpret, explore and move from an algebraic representation to its geometric expression.

Duval's model of semiotic representations was an important support in the design and development of the proposal. Based on the perception of the students in the opinion survey, it can be said that the sequence of indications in the workbook, which included the use of technology to promote student learning, helped center their attention on the logical relationship of the various representations, and encouraged discussion about the mathematical objects they were discovering.

The area calculation topic contains several representation records and these can be interpreted by a vertically simple or horizontally simple integral. From the analysis of the data in the study it was possible to verify that the identification of these different forms of representation

and the transit through them, favored conceptualization by students such as affirms it Duval (2006).

The use of technology favored the exploration and representation of the areas so that the students could:

- 1) Properly establish certain regions between curves in the plane.
- 2) Pose inequalities and define iterated integrals associated with these regions.
- 3) Obtain the value of the area of the built region.

For future research, it would be necessary to rethink the application time of the same, since it was the only aspect in which several students, approximately the 20%, they considered that it was not enough to complete the activities in the workbook.

Finally, from the results obtained in this work, it can be affirm that ICT can be used to improve the teaching and learning processes of mathematical objects, although its use in the classroom requires adequate planning. Technology has come to positively transform the way of teach and learn math.

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## Appendix

### OPINION SURVEY ON ACTIVITIES

**Instructions:** Based on experience during the sessions, mark an x in the box that corresponds to the following questions. Please be HONEST as the survey is anonymous. Consider the most appropriate from the development of the activities carried out.

#### Observation scale

1	2	3	4	5
Completely disagree	Disagree	Is indifferent to me	Agree	Completely agree

Opinions					
	1	2	3	4	5
1. The vision of the topic was improved with the development of the activities.					
2. The activities carried out were interesting.					
3. The use of technology motivates interest in learning the topic.					
4. The use of the programs facilitates the interpretation of graphs.					
5. The time dedicated to the activities you consider was sufficient.					
6. You consider that the methodology used in each of the sessions was adequate.					
7. I like doing the activities in the workbook.					
8. The instructions in the workbook were clear and understandable.					
9. The use of the workbook was clear and structured.					
10. The activities carried out encouraged the learning of the topic.					
11. With the activities the topic was covered according to the thematic content.					
12. The support provided by the teacher was adequate.					
13. The support provided by the teacher was adequate.					
14. The teacher's performance for the development of the topic was adequate.					
15. The teacher managed to develop a positive work					



environment (respect, openness to dialogue, feedback).					
16. You consider that, in general, the material used for the topic was of quality.					

From what you learned in class on the topic of calculating areas between two curves, what aspects they called your attention?

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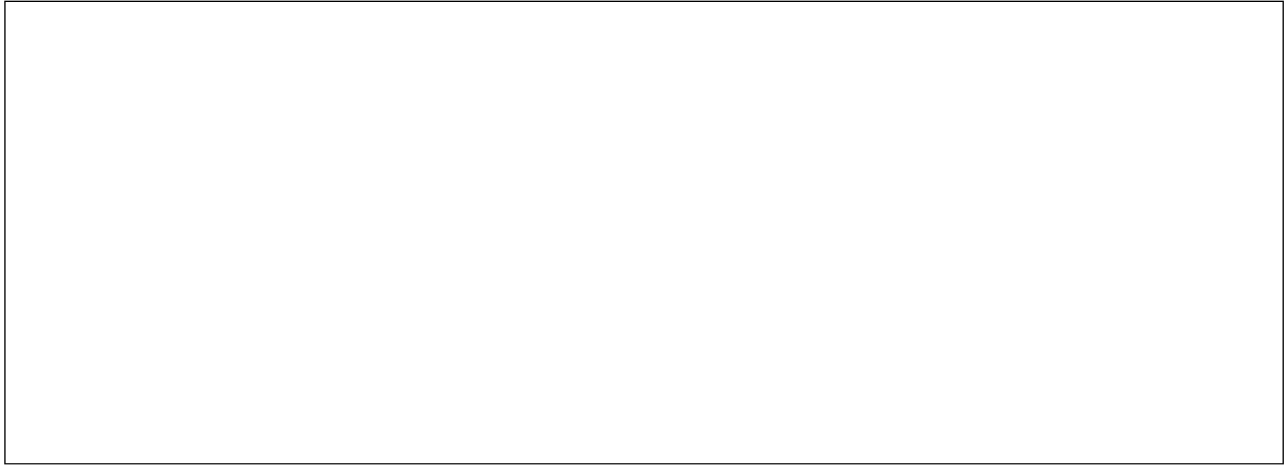
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In general, give your opinion on the activities carried out, the use of the software and the performance of the teachers.

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**Write a comment that is not covered in the previous questions and that is related to the quality of the material used or the way it was presented or used.**



**THANK YOU FOR YOUR COOPERATION**